

# Quantum speed limits for Bell-diagonal states

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Bounds of the minimum evolution time between two distinguishable states of a system can help to assess the maximal speed of quantum computers and communication channels. We study the quantum speed limit time of a composite quantum states in the presence of nondissipative decoherence. For the initial states with maximally mixed marginals, we obtain the exactly expressions of quantum speed limit time which mainly depend on the parameters of the initial states and the decoherence channels. Furthermore, by calculating quantum speed limit time for the time-dependent states started from a class of initial states, we discover that the quantum speed limit time gradually decreases in time, and the decay rate of the quantum speed limit time would show a sudden change at a certain critical time. Interestingly, at the same critical time, the composite system dynamics would exhibit a sudden transition from classical to quantum decoherence.

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## I. INTRODUCTION

Quantum mechanics establishes the fundamental bounds for the minimum evolution time between two states of a given system. Bounds of this evolution time, known as quantum speed limit time (QSLT), are intimately related to the evolution speed of quantum systems. The applications of these bounds are shown in different scenarios, including quantum communication [1], quantum metrology [2], quantum computation [3], as well as quantum optimal control protocols [4]. Derivations of these basic bounds focused on the closed system with unitary evolution, are obtained by Mandelstam-Tamm (MT) type bound and Margolus-Levitin (ML) type bound [5–16]. Moreover, the extensions of the QSLT for nonunitary evolution of open systems are also studied [17–20], since the relevant influence of the environment on processing or information transferring systems can not be ignored. It is shown that two unified bounds of QSLT for the pure or mixed initial states including both MT and ML types for nonunitary dynamics process have been formulated, respectively [19, 20]. Although these two unified bounds have already been used to illustrate the quantum evolution speed for a qubit state (whether it is a pure state or mixed state) under decoherence process. As we all know that the correlated composite quantum states are playing a central role in quantum information and communication theory [21, 22]. Such as, in order to fulfil a long-distance high-fidelity quantum communication in a noisy channel, entanglement of a composite system (two distant sites) must be used to implement quantum repeaters [23]. And the QSLT on the evolu-

tion of an open composite quantum system would help to deal with the robustness of quantum simulators and computers against decoherence [24]. So the application of these unified bounds to assess the evolution speed of the arbitrary states in a composite quantum system with nonunitary dynamics, is very much looking forward.

In this paper, we are interested in the evolution speed of two-qubit Bell-diagonal states under certain noise channels (i.e., phase flip, bit flip, and bit-phase flip). By making use of the unified bound of QSLT for arbitrary mixed states [20], we have identified the conditions on the different expressions of the QSLT for the initial two-qubit Bell-diagonal states under decoherence process. In the following, we also focus on the QSLT for the time-dependent states which started from the initial Bell-diagonal states in the whole dynamics process. We demonstrate that the QSLT will be reduced gradually with the starting point in time under the noise channels, that is to say, the two-qubit system executes a speeded-up dynamics evolution process. Moreover, for certain initial Bell-diagonal states considered in Ref. [25], a remarkable result we find in the paper is the existence of a sudden change of the decay rate of the QSLT at a certain critical time  $\tau_c$ . And at the same critical time  $\tau_c$ , a sudden transition from classical to quantum decoherence for this class of initial Bell-diagonal states also appears. Then we can specifically point out that the evolution speed of the whole dynamics process can be described qualitatively by the classical decoherence for  $\tau < \tau_c$ , and quantum decoherence for  $\tau > \tau_c$ . Finally, we also recognize a symmetry among the above three decoherence channels.

This paper is organized as follows. In Sec. II, we first review the definition of the QSLT, and we give the explicit formulas of the QSLT for the two-qubit initial Bell-diagonal states under certain noise channels. In Sec. III, we apply the QSLT of the time-dependent states to illustrate the speed of the whole dynamics evolutionary process. We conclude in the last section.

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## II. QUANTUM SPEED LIMIT UNDER DECOHERENCE CHANNELS

Firstly, we start with the definition of the QSLT for open quantum systems. A unified lower bound, including both MT and ML types, has been derived in Refs. [19, 20]. With the help of the von Neumann trace inequality and the Cauchy-Schwarz inequality, the QSLT between an arbitrary initially mixed state  $\rho_0$  and its target state  $\rho_{\tau_D}$ , governed by the master equation  $\dot{\rho}_t = L_t \rho_t$ , with  $L_t$  the positive generator of the dynamical semigroup, is as follows

$$\tau_{QSL} = \max\left\{\frac{1}{\sum_{i=1}^n \sigma_i \varrho_i}, \frac{1}{\sqrt{\sum_{i=1}^n \sigma_i^2}}\right\} B(\rho_0, \rho_{\tau_D}), \quad (1)$$

with  $\overline{X} = \tau_D^{-1} \int_0^{\tau_D} X dt$ ,  $B(\rho_0, \rho_{\tau_D}) = |\text{tr}(\rho_0 \rho_{\tau_D}) - \text{tr}(\rho_0^2)|$  denotes the Bures distance between the initial state  $\rho_0$  and the target state  $\rho_{\tau_D}$ , and  $\sigma_i$  are the singular values of  $\dot{\rho}_t$  and  $\varrho_i$  those of the initial mixed state  $\rho_0$ . For a pure initial state  $\rho_0 = |\phi_0\rangle\langle\phi_0|$ , the singular value  $\varrho_i = \delta_{i,1}$ , then the expression of  $\tau_{QSL}$  thus recovers the unified bound of the QSLT given in Ref. [19]. So the QSLT formulated in (1) can effectually defines the minimal evolution time for arbitrary initial states, and also be used to deal with the maximal speed of evolution of open quantum systems.

We consider two qubits without mutual interaction, each one individually coupled to its own non-dissipative noisy channels. Here, we mainly focus on the phase flip, bit flip, and bit-phase flip channels. The dynamics of each qubit is governed by a master equation that gives rise to a completely positive trace-preserving map  $\varepsilon(\cdot)$ . In the Born-Markov approximation, the operator-sum representation is given by  $\varepsilon(\rho_{AB}) = \sum_{i,j} K_i^A K_j^B \rho_{AB} K_j^{B\dagger} K_i^{A\dagger}$ , with  $K_i^{A(B)}$  are the Kraus operators that describe the noise channels  $A$  and  $B$ . In order to clear the QSLT of the two-qubit initial mixed states in the above noisy channels, we take two-qubit Bell-diagonal state as the initial state of the composite system, described by

$$\rho_0 = \frac{1}{4}(\mathbb{I}_A \otimes \mathbb{I}_B + \sum_{i=1}^3 c_i \sigma_i^A \otimes \sigma_i^B), \quad (2)$$

where  $c_i \in \mathbb{R}$  such that  $0 \leq |c_i| \leq 1$  for  $i = 1, 2, 3$ , and  $\mathbb{I}_{A(B)}$  is the identity operator in subspace  $A(B)$ . The state in Eq. (2) includes the Werner states ( $|c_1| = |c_2| = |c_3| = c$ ) and the Bell states ( $|c_1| = |c_2| = |c_3| = 1$ ).

We first focus on the phase flip channel. For this channel, the Kraus operators are given by [21, 26]  $K_0^A = \text{diag}(\sqrt{1-p_A/2}, \sqrt{1-p_A/2}) \otimes \mathbb{I}_B$ ,  $K_1^A = \text{diag}(\sqrt{p_A/2}, \sqrt{p_A/2}) \otimes \mathbb{I}_B$ ,  $K_0^B = \mathbb{I}_A \otimes \text{diag}(\sqrt{1-p_B/2}, \sqrt{1-p_B/2})$ ,  $K_1^B = \mathbb{I}_A \otimes \text{diag}(\sqrt{p_B/2}, \sqrt{p_B/2})$ , here,  $p_{A(B)}$  is used as parametric time in channel  $A(B)$ , and by considering the symmetric situation in which the decoherence rate is

equal in both channels, so  $p_A = p_B$ . For the initial state of Eq. (2), the time evolution of the total system is

$$\begin{aligned} \rho(t) = & \lambda_{\Psi}^+(t) |\Psi^+\rangle\langle\Psi^+| + \lambda_{\Phi}^+(t) |\Phi^+\rangle\langle\Phi^+|, \\ & + \lambda_{\Psi}^-(t) |\Psi^-\rangle\langle\Psi^-| + \lambda_{\Phi}^-(t) |\Phi^-\rangle\langle\Phi^-|, \end{aligned} \quad (3)$$

where

$$\begin{aligned} \lambda_{\Psi}^{\pm}(t) &= (1 \pm c_1(t) \mp c_2(t) + c_3(t))/4, \\ \lambda_{\Phi}^{\pm}(t) &= (1 \pm c_1(t) \pm c_2(t) - c_3(t))/4, \end{aligned} \quad (4)$$

and  $|\Psi^{\pm}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ ,  $|\Phi^{\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$  are the four Bell states. The time dependent coefficients in Eq. (4) are  $c_1(t) = (1-p)^2 c_1$ ,  $c_2(t) = (1-p)^2 c_2$ , and  $c_3(t) = c_3$ , with  $p = 1 - \exp(-\gamma t)$ , and  $\gamma$  the phase damping rate.

Now, we calculate the QSLT for the initial Bell-diagonal state  $\rho_0$  under the phase flip channel. According to the expression (1), we can clearly find that the Bures distance between  $\rho_0$  and  $\rho_{\tau_D}$ ,  $B(\rho_0, \rho_{\tau_D}) = \frac{1}{4}(p_{\tau_D}^2 - 2p_{\tau_D})(|c_1|^2 + |c_2|^2)$ . Thus our main task is to calculate the singular values of  $\rho_0$  and  $\dot{\rho}_t$ , respectively. For  $\dot{\rho}_t$ , the singular values  $\sigma_i$  are

$$\begin{aligned} \sigma_1 &= \sigma_2 = \frac{1}{2} |\dot{p}(1-p)(|c_1| + |c_2|)|, \\ \sigma_3 &= \sigma_4 = \frac{1}{2} |\dot{p}(1-p)(|c_1| - |c_2|)|. \end{aligned} \quad (5)$$

While for the initial state  $\rho_0$ , the singular values  $\varrho_i$  depend on the region of the coefficients  $c_i$ . Then, in the following, we divide the region of the coefficients  $c_i$  into four parts.

Case I: If  $|c_3| \geq |c_1| \geq |c_2|$  in Eq. (2), the singular values  $\varrho_i$  of the initial states  $\rho_0$  satisfy  $\varrho_1 + \varrho_2 = \frac{1}{2}(1 + |c_3|)$ ,  $\varrho_3 + \varrho_4 = \frac{1}{2}(1 - |c_3|)$ . In this case,  $\sum_{i=1}^4 \sigma_i \varrho_i = \frac{1}{2}(1-p)|\dot{p}|(|c_1| + |c_2||c_3|)$  is always less than  $\sqrt{\sum_{i=1}^4 \sigma_i^2} = (1-p)|\dot{p}|\sqrt{|c_1|^2 + |c_2|^2}$ , so the QSLT for the class of initial states in this case can be obtained  $\tau_{QSL}^I = \frac{\tau_D(|c_1|^2 + |c_2|^2)}{|c_1| + |c_2||c_3|}$ .

Case II: If  $|c_3| \geq |c_2| \geq |c_1|$ , we obtain  $\sum_{i=1}^4 \sigma_i \varrho_i = \frac{1}{2}(1-p)|\dot{p}|(|c_2| + |c_1||c_3|)$ , the ML type bound on the QSLT is also tight for the initial states in the case II, then we acquire  $\tau_{QSL}^{II} = \frac{\tau_D(|c_1|^2 + |c_2|^2)}{|c_2| + |c_1||c_3|}$ .

Case III: When  $|c_1| \geq |c_3|, |c_2|$ , the singular values  $\varrho_i$  are given by  $\varrho_1 + \varrho_2 = \frac{1}{2}(1 + |c_1|)$ ,  $\varrho_3 + \varrho_4 = \frac{1}{2}(1 - |c_1|)$ , and  $\sum_{i=1}^4 \sigma_i \varrho_i = \frac{1}{2}(1-p)|\dot{p}||c_1|(1 + |c_2|)$  is also less than  $\sqrt{\sum_{i=1}^4 \sigma_i^2}$ . The QSLT of the initial states in case III can be written  $\tau_{QSL}^{III} = \frac{\tau_D(|c_1|^2 + |c_2|^2)}{|c_1|(1 + |c_2|)}$ . The QSLT does not depend on the coefficient  $c_3$ .

Case IV: Finally, if  $|c_2| \geq |c_3|, |c_1|$ , we acquire  $\varrho_1 + \varrho_2 = \frac{1}{2}(1 + |c_2|)$ ,  $\varrho_3 + \varrho_4 = \frac{1}{2}(1 - |c_2|)$ , and  $\sum_{i=1}^4 \sigma_i \varrho_i = \frac{1}{2}(1-p)|\dot{p}||c_2|(1 + |c_1|)$ . With this, it is easy to show that the QSLT in this case yields  $\tau_{QSL}^{IV} = \frac{\tau_D(|c_1|^2 + |c_2|^2)}{|c_2|(1 + |c_1|)}$  (also independent of the coefficient  $c_3$ ).

Inspection of the expressions  $\tau_{QSL}^I$ ,  $\tau_{QSL}^{II}$ ,  $\tau_{QSL}^{III}$  and  $\tau_{QSL}^{IV}$  shows that in all four cases the QSLT mainly depends on the coefficients  $c_i$  of the initial state  $\rho_0$ . Particularly, for all states in cases  $|c_3| \geq |c_2|, |c_1|$  (cases I and II), the QSLT becomes inversely proportional to  $|c_3|$ . While for all states in which the coefficients  $c_i$  belong to cases III and IV, the QSLT is independent of  $c_3$ . In order to investigate the effects of different values of the coefficients  $c_i$  on the QSLT under phase flip channel, we plot the QSLT of the initial Bell-diagonal states  $\rho_0$  as a function of  $|c_1|$  and  $|c_2|$  for a given choice  $|c_3| = 0.4$ , in Fig. 1. And then, two remarkable features can be acquired from Fig. 1: (i) The QSLT is symmetrical with respect to the line  $|c_1| = |c_2|$ . That is to say, the states  $\rho_0(|c_1|, |c_2|, |c_3|)$  and  $\rho_0(|c_2|, |c_1|, |c_3|)$  have the same quantum speed of the evolution under phase flip channel. (ii) The QSLTs for the initial states in cases I and II are always smaller than for those states in cases III and IV. So we note that, under the local phase flip channel, the Bell-diagonal states in cases I and II have smaller minimal evolution time than those in cases III and IV.

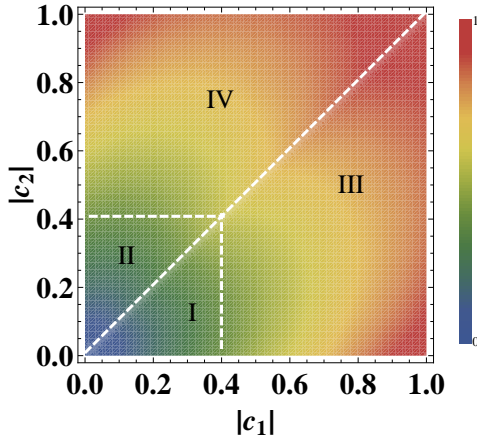


FIG. 1: (Color online) The QSLT under phase flip channel as a function of the coefficients  $|c_1|$  and  $|c_2|$  with  $|c_3| = 0.4$ , here the driving time  $\tau_D = 1$ . The dashed white lines divide the entire range of  $|c_1|$  and  $|c_2|$  into four parts: case I,  $|c_3| \geq |c_1| \geq |c_2|$ ; case II,  $|c_3| \geq |c_2| \geq |c_1|$ ; case III,  $|c_1| \geq |c_3|, |c_2|$ ; and case IV,  $|c_2| \geq |c_3|, |c_1|$ .

For the bit flip channel, the Kraus operators are [21, 26]  $K_0^A = \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes \mathbb{I}_B$ ,  $K_1^A = \sqrt{p/2}\sigma_x^A \otimes \mathbb{I}_B$ ,  $K_0^B = \mathbb{I}_A \otimes \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$ ,  $K_1^B = \mathbb{I}_A \otimes \sqrt{p/2}\sigma_x^B$ . The eigenvalue spectrum of  $\rho_{AB}(t)$  is also given by Eq. (4), where the time dependent coefficients are  $c_1(t) = c_1$ ,  $c_2(t) = (1-p)^2 c_2$ , and  $c_3(t) = (1-p)^2 c_3$ . The QSLT of the Bell-diagonal state  $\rho_0$  under bit flip channel is symmetrical to that for the phase flip channel. We just exchange the position of  $c_1$  and  $c_3$  in all expressions for the phase flip channel, so the

QSLT under the bit flip channel can be obtained

$$\tau_{QSL}^B = \begin{cases} \frac{\tau_D(|c_2|^2 + |c_3|^2)}{|c_3| + |c_1||c_2|}, & |c_1| \geq |c_3| \geq |c_2| \\ \frac{\tau_D(|c_2|^2 + |c_3|^2)}{|c_2| + |c_1||c_3|}, & |c_1| \geq |c_2| \geq |c_3| \\ \frac{\tau_D(|c_2|^2 + |c_3|^2)}{|c_3|(1+|c_2|)}, & |c_3| \geq |c_1|, |c_2| \\ \frac{\tau_D(|c_2|^2 + |c_3|^2)}{|c_2|(1+|c_3|)}, & |c_2| \geq |c_1|, |c_3| \end{cases}. \quad (6)$$

For  $|c_3| \geq |c_1|, |c_2|$  and  $|c_2| \geq |c_1|, |c_3|$ , under bit flip channel, the QSLT of the initial state does not depend on  $c_1$ . And the QSLT is symmetrical with respect to the line  $|c_2| = |c_3|$ , i.e.,  $\tau_{QSL}^B(\rho_0(|c_1|, |c_2|, |c_3|)) = \tau_{QSL}^B(\rho_0(|c_1|, |c_3|, |c_2|))$ . Furthermore, the Bell-diagonal states in cases  $|c_1| \geq |c_3| \geq |c_2|$  and  $|c_1| \geq |c_2| \geq |c_3|$  have the smaller QSLTs than those in cases  $|c_3| \geq |c_1|, |c_2|$  and  $|c_2| \geq |c_1|, |c_3|$ .

Finally, for the bit-phase flip channel, the Kraus operators can be shown [21, 26]  $K_0^A = \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes \mathbb{I}_B$ ,  $K_1^A = \sqrt{p/2}\sigma_y^A \otimes \mathbb{I}_B$ ,  $K_0^B = \mathbb{I}_A \otimes \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$ ,  $K_1^B = \mathbb{I}_A \otimes \sqrt{p/2}\sigma_y^B$ . In this noise channel, the time dependent coefficients are  $c_1(t) = (1-p)^2 c_1$ ,  $c_2(t) = c_2$ , and  $c_3(t) = (1-p)^2 c_3$ . Once more, by swapping  $c_2$  and  $c_3$  in the phase flip channel, we can obtain

$$\tau_{QSL}^{B-P} = \begin{cases} \frac{\tau_D(|c_1|^2 + |c_3|^2)}{|c_3| + |c_2||c_1|}, & |c_2| \geq |c_3| \geq |c_1| \\ \frac{\tau_D(|c_1|^2 + |c_3|^2)}{|c_1| + |c_2||c_3|}, & |c_2| \geq |c_1| \geq |c_3| \\ \frac{\tau_D(|c_1|^2 + |c_3|^2)}{|c_1|(1+|c_3|)}, & |c_1| \geq |c_2|, |c_3| \\ \frac{\tau_D(|c_1|^2 + |c_3|^2)}{|c_3|(1+|c_1|)}, & |c_3| \geq |c_2|, |c_1| \end{cases}. \quad (7)$$

The fact that, for  $|c_1| \geq |c_2|, |c_3|$  and  $|c_3| \geq |c_2|, |c_1|$ , the coefficient  $c_2$  would not affect the QSLT under the bit-phase flip channel. It is also simple to see that, for the bit-phase flip channel,  $\tau_{QSL}^{B-P}(\rho_0(|c_1|, |c_2|, |c_3|)) = \tau_{QSL}^{B-P}(\rho_0(|c_3|, |c_2|, |c_1|))$ . And the QSLTs for the initial states in cases  $|c_2| \geq |c_3| \geq |c_1|$  and  $|c_2| \geq |c_1| \geq |c_3|$  are always smaller than for those states in cases  $|c_1| \geq |c_2|, |c_3|$  and  $|c_3| \geq |c_2|, |c_1|$ .

It is worth mentioning that the QSLT of any Bell-diagonal states under phase flip, bit flip, and bit-phase flip channels, can be accurately calculated. This is one of our main results in this work. Particularly, for the initial Werner state ( $|c_1| = |c_2| = |c_3| = c$ ), the QSLTs under these three noise channel are all equal to  $\frac{2\tau_D}{1+1/c}$ , which is proportional to  $c$ .

### III. QUANTUM EVOLUTION SPEED FOR OPEN DYNAMICS PROCESS

In what follows, we shall illustrate the application of the QSLT to the quantum evolution speed of the whole dynamics process under different decoherence channels.

The QSLT can demonstrate the speed of the dynamics evolution from a time-evolution state  $\rho_\tau$  to another  $\rho_{\tau+\tau_D}$  by a driving time  $\tau_D$ . It is easy to find that the time-evolution state  $\rho_\tau$  at any point in time also maintains the form of Bell-diagonal state. So the QSLT starting from an arbitrary time-evolution state  $\rho_\tau$  can be calculated by the expressions of  $\tau_{QSL}$  in Section II. We would examine the evolution speed of the whole dynamics process where the system starts from the two-qubit Bell-diagonal state in Eq. (2), under phase flip, bit flip and bit-phase flip channels, respectively.

In the first place, under the phase flip channel,  $|c_3| \geq |c_1|, |c_2|$  is chosen for  $\rho_0$ . During the dynamics evolution process,  $c_3$  remains unchanged, so the condition  $|c_3| \geq |c_1(\tau)|, |c_2(\tau)|$  is fulfilled in  $\tau \in [0, \infty)$ . By considering an arbitrary mixed state  $\rho_\tau$  evolves to another  $\rho_{\tau+\tau_D}$  under a driving time  $\tau_D$ , the QSLT can be calculated

$$\tau_{QSL}^P = \begin{cases} \frac{\tau_D(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_1|+|c_2||c_3|}, & |c_1| \leq |c_2| \\ \frac{\tau_D(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_2|+|c_1||c_3|}, & |c_1| \geq |c_2| \end{cases}. \quad (8)$$

From Eq. (8), one sees immediately that, the QSLT for the time-dependent state  $\rho_\tau$  decays monotonically, that is to say, in this case, the whole evolution of the two-qubit Bell-diagonal state can exhibits a speeded-up process under the local phase flip noise channels.

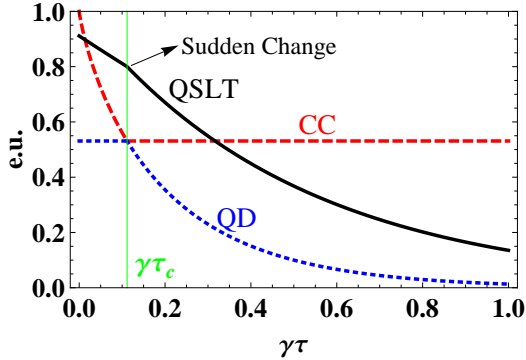


FIG. 2: (Color online) The QSLT (solid black line), classical correlation (CC, dashed red line) and quantum correlation (QD, dotted blue line) under local phase flip channels as a function of the initial time parameter  $\gamma\tau$ . We choose the class of initial Bell-diagonal states considered in Ref. [25], in which  $c_1 = \pm 1$ ,  $c_2 = \mp c_3$ , here  $c_1 = 1$ ,  $c_2 = -0.8$ , and  $c_3 = 0.8$ , and the driving time  $\tau_D = 1$ . On this initial condition, the behavior of sudden transition from classical to quantum decoherence occurs at  $\tau = \tau_c$ . The sudden change of the decay of QSLT has been marked in this figure.

However, if  $\rho_0$  satisfies  $|c_1| \geq |c_2|, |c_3|$  or  $|c_2| \geq |c_1|, |c_3|$ , due to  $c_3$  independence of the evolution time, it is easy to find that, for  $\tau \leq \tau_c$ , with  $\tau_c = \frac{1}{2} \ln(\frac{\max\{|c_1|, |c_2|\}}{|c_3|})$ , the conditions  $|c_1(\tau)| \geq |c_2(\tau)|, |c_3|$  or  $|c_2(\tau)| \geq |c_1(\tau)|, |c_3|$  are always true. By considering the dynamics process  $\tau \in [0, \tau_c]$ , we can calculate the QSLT for the qubits to

evolve from  $\rho_\tau$  to  $\rho_{\tau+\tau_D}$ ,

$$\tau_{QSL}^P = \begin{cases} \frac{\tau_D(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_1|[1+(1-p_\tau)^2|c_2|]} , & |c_1| \geq |c_2|, |c_3| \\ \frac{\tau_D(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_2|[1+(1-p_\tau)^2|c_1|]} , & |c_2| \geq |c_1|, |c_3| \end{cases}. \quad (9)$$

On the other hand, for  $\tau > \tau_c$ , then  $|c_3| \geq |c_1(\tau)| \geq |c_2(\tau)|$  or  $|c_3| \geq |c_2(\tau)| \geq |c_1(\tau)|$ , so when  $\tau \in (\tau_c, \infty)$ , the expressions of QSLT can be obtained by Eq. (8). Hence, in the conditions  $|c_1| \geq |c_2|, |c_3|$  or  $|c_2| \geq |c_1|, |c_3|$ , we note that, the decay rate of the QSLT changes suddenly at  $\tau = \tau_c$ . So we can conclude that, in the above case, the open system executes a speeded-up dynamics evolution process, but the increasing rate of the evolution speed would have a sudden change at  $\tau = \tau_c$ . Let us, finally, study the QSLT, classical correlation and quantum correlation (quantum discord) on the class of initial states for which  $c_1 = \pm 1$  and  $c_2 = \mp c_3$ , with  $|c_3| < 1$ , in Fig. 2. It is worth noting that, as noted in Ref. [25], for this initial condition, the behavior of sudden transition from classical to quantum decoherence can occur at  $\tau = \tau_c$ . And at this time point, the decay rate of the QSLT can also change suddenly. Remarkably, the attenuations of the QSLT and classical correlation are synchronous in time until  $\tau = \tau_c$ . However, for  $\tau > \tau_c$ , the synchronous attenuation behavior occurs between the QSLT and quantum discord. This means that the speed of the dynamics evolution from a time-evolution state  $\rho_\tau$  to another  $\rho_{\tau+\tau_D}$  can be clearly signatored by the classical correlation of the time-evolution state  $\rho_\tau$  in the case  $\tau < \tau_c$ . While for  $\tau > \tau_c$ , we can use the quantum discord of  $\rho_\tau$  to describe the speed of dynamics process, qualitatively. This is a newly noticed phenomenon.

Finally, when we consider the bit flip and the bit-phase flip channels, the behaviors of the evolution speed of the whole dynamics process under the phase flip channel described above can also occur under other conditions on the  $c_i$  in state (2). It is simple to see that, for the bit flip and bit-phase flip channels, the QSLT starting from an arbitrary time-evolution state  $\rho_\tau$  to  $\rho_{\tau+\tau_D}$  can be obtained with  $c_1$  and  $c_2$  replacing  $c_3$ , respectively.

#### IV. CONCLUSION

In summary, based on the unified bound of QSLT for arbitrary mixed states, we have calculated the expressions of the QSLT to characterize the speed of evolution for an open composite quantum system. In particular, for the initial two-qubit Bell-diagonal state, we have identified four different expressions of the QSLT under decoherence, which depend on the coefficients of the initial composite state and on the noise channel. Moreover, by studying the bound for the arbitrary time-dependent states started from a class of two-qubit states cited in Ref. [25], a remarkable feature has been demonstrated that the decay rate of the QSLT can change suddenly at a critical time  $\tau_c$ , at this moment the behavior of sudden

transition from classical to quantum decoherence also occurs. This evolution speed of the dynamics process plays an essential role in the understanding of the classical-to-quantum decoherence transition. Our results may be of interests in exploring the speed of quantum computation and quantum information processing in the presence of noise.

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